Analytical Solution for Constant Enthalpy MHD Accelerator

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THE Aerodynamics and Propulsion Research Laboratory of Aerospace Corporation is studying the feasibility of a low-density hypervelocity wind tunnel employing a constant-enthalpy MHD accelerator In this facility, high-temperature seeded air is expanded isentropically to supersonic velocity, is accelerated at constant enthalpy by a crossed field accelerator, and is then again expanded isentropically to a test section Theoretically, the facility provides almost exact duplication of hypervelocity re-entry conditions at high altitudes A facility of this type has been proposed by Boison and Ring [1]

In the initial phases of the Aerospace Corporation investigation, it was desirable to have a simple closed-form analytic solution for the constant-enthalpy MHD accelerator in order to make a parametric study of the wind-tunnel facility Although several closed-form solutions exist (e g , Refs 2 and 3), none seemed suitable The existing solutions and their first modifications yielded accelerators that were either too long, had too large an area ratio, or had a major portion of the energy addition, or area change, occur over a relatively short portion of the accelerator The following solution was then obtained which seemed to overcome these shortcomings In this solution, the axial energy addition and area variation are specified, in addition to the assumption of constant enthalpy

Consider one-dimensional MHD flow in a channel, as indicated in Fig 1 The superscript asterisk indicates a dimensional quantity The applied electric field is E_y^*, and the applied magnetic field is B^* The electrodes are assumed to be segmented so that the current j^* is in the y^* direction, regardless of the value of the Hall parameter $(\omega_e\tau_e)^*$ An axial electrical field E_x^* is induced, for $(\omega\ \tau\ )^* \neq 0$, which will be found later Let subscript 1 indicate conditions at the accelerator inlet ($x = 0$), and define the following nondimensional variables:

$$\begin{aligned} \rho &= (\rho/\rho_1)^* & B &= (B/B_1)^* \\ p &= (p/\rho_1 u_1^2)^* & E_y &= (E_y/u_1B_1)^* \\ h &= (h/u_1^2)^* & j &= (j/\sigma_1 u_1 B_1)^* \\ u &= (u/u_1)^* & \sigma &= (\sigma/\sigma_1)^* \\ A &= (A/A_1)^* & x &= (x/L)^* \end{aligned} \tag{1}$$

where $L^* \equiv (\rho_1 u_1/\sigma_1 B_1^2)^*$ Note that $p_1 = (p_1/\rho_1 u_1^2)^*$ and $= 1/\gamma_1 M_1^2$ for an ideal gas The equations of motion, neglecting heat conduction, viscous effects, and ion slip,[4] become, in mks units, the following:

Energy

$$\rho u \frac{dh}{dx} = u\frac{dp}{dx} + \frac{j^2}{\sigma} \tag{2a}$$

Momentum

$$\rho u \frac{du}{dx} = -\frac{dp}{dx} + jB \tag{2b}$$

Continuity

$$\rho u A = 1 \tag{2c}$$

Ohms Law

$$j = \sigma[E_y - uB] \tag{2d}$$

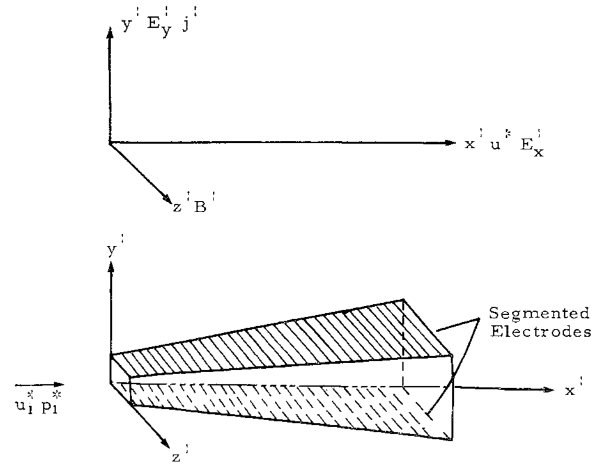

Fig 1 Cooıdinate system vaiiables, and channel

State

$$p = p(\rho,h) \tag{2e}$$

Scalar Conductivity Law

$$\sigma = \sigma(\rho,h) \tag{2f}$$

Ion slip effects are negligible when[4] $(\omega\ \tau\ )^*(\omega_i\tau_i)^* \ll 1$ The latter is satisfied[4] for values of $(\omega\ \tau\ )^*$ up to about 10, since $(\omega\ \tau\ /\omega_i\tau_i)^* \approx 100$ to 1000 Equations (2a–2d) can be combined to yield

$$(d/dx)[h + (u^2/2)] = jE_yA \tag{3}$$

Closed-form solutions are obtainable for a variety of cases For the present purposes, assume that

$$h = h_1 \tag{4a}$$

$$jE_yA = \phi_1 u^n \tag{4b}$$

$$A = (1 + \alpha\phi_1 x)^2 \tag{4c}$$

where ϕ_1, n, and α are constants Here Eq (4a) is the constant enthalpy assumption, Eq (4b) specifies the energy addition (per unit x) as a function of u, and Eq (4c) specifies the area variation to be equivalent to that for a truncated cone Also assume that $\rho = p/p_1$ (perfect gas) and $\sigma = 1$ (constant conductivity) The resulting solution is

$$\begin{aligned} x &= (u^{2-n} - 1)/(2-n)\phi_1 \quad \text{for } n \neq 2 \\ &= (\ln u)/\phi_1 \qquad\qquad n = 2 \end{aligned} \tag{5a}$$

$$\rho = p/p_1 = 1/uA \tag{5b}$$

$$j^2 = p_1\phi_1(u^{n-2} + 2\alpha/A^{1/2})/A \tag{5c}$$

$$E_y = \phi_1 u^n/jA \tag{5d}$$

$$B = (E_y - j)/u \tag{5e}$$

where

$$\phi_1 = E_{y\,1}(E_{y\,1} - 1) \tag{6a}$$

$$\alpha = \tfrac{1}{2}[(E_{y\,1} - 1)/(p_1E_{y\,1}) - 1] \tag{6b}$$

The constant α must have the value indicated in Eq (6b) in order to satisfy Eq (5c) at $x = 0$ A more general solution is indicated in the Appendix

The axial electric field and potential variation, because of the Hall effect, can also be found For a slightly ionized gas ($\lesssim 1\%$) in the temperature range 4000° to 8000°K, Ref 2 states

$$(\omega\ \tau\ )^* \approx 10^{-1}B^*\rho_{st}^*/\rho^*$$

where ρ_{st}^* is the gas density at standard temperature and pressure, and B^* is in webers per square meter Thus, the variation of the Hall parameter is given by

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$$\omega\,\tau_e \equiv \frac{(\omega_e \tau_e)^*}{(\omega\,\tau\,)_1^*} = \frac{B}{\rho} \tag{7}$$

Since it was assumed that segmented electrodes were used, such that j^* was in the y^* direction only, an axial electric field $E_x{}^*$ is induced which can be found from[4]

$$E_x{}^* = (\omega\,\tau_e)^* j^*/\sigma^*$$

where ion slip is again neglected In nondimensional variables,

$$E_x \equiv \left[\frac{E_x}{u_1 B_1(\omega\,\tau\,)_1}\right]^* = \frac{Bj}{\rho\sigma}$$

$$= \frac{Bj}{\rho} \quad \text{for } \sigma = 1 \tag{8}$$

The net potential difference between $x^* = 0$ and x^* is

$$\Phi^* = -\int_0^{x^*} E_x{}^* dx^*$$

or

$$\Phi \equiv \frac{\Phi^*}{[u_1 B_1(\omega\,\tau\,)_1 L]^*} = -\int_0^x E_x dx = p_1 \ln\left(\frac{1}{\rho}\right) - \left(\frac{u^2 - 1}{2}\right) \tag{9}$$

The upstream and downstream ends of the accelerator must be electrically insulated from each other in order to prevent a current flow in the x direction

Equations (4–9) define the present solution There are three free parameters in the nondimensional equations: $E_{y\,1}$, n, and p_1 If the inlet flow conditions to the accelerator are specified, p_1 is defined, and only the parameters $E_{y\,1}$ and n remain The length x, required to achieve a velocity u, decreases as $E_{y\,1}$ and n increase However, it is desirable to have the values of n and $(E_{y\,1} - 1)$ as near to zero as possible without incurring excessive accelerator length The case $n = 0$ corresponds to uniform energy addition per unit x, and all portions of the accelerator are equally important As n increases, the downstream portions of the accelerator add proportionately more energy (and may do so inefficiently) Values of $E_{y\,1}$ near 1 correspond to relatively little joule heating As $E_{y\,1}$ increases, the fraction of the input energy which goes into joule heating also increases, and the downstream channel area increases (in order to keep the enthalpy constant) This causes lower p,ρ (which limits the tunnel altitude simulation) and higher $\omega\,\tau$ (which may lead to ion slip) at the downstream stations Numerical evaluation of Eqs (4–9) for various values of n and $E_{y\,1}$ will indicate the appropriate values for reasonable accelerator length, $\omega\,\tau$, and tunnel altitude simulation

The present solution requires that E_y and B vary with x, but this should not cause difficulty in an experimental facility The use of segmented electrodes permits E_y to be varied at will Similarly, the spacing between the sides of an air core magnet can be varied to provide the correct variation of B with x Equations (4–9) have been used in a preliminary design of a pulsed-type hypervelocity wind tunnel This design will be reported in a later publication

Appendix

The present solution can be readily generalized to the case where h and jE_yA are specified functions of u and A is a specified function of x,u The equation of state [Eq (2e)] and the scalar conductivity law [Eq (2f)] can also be arbitrarily specified The relation between x and u becomes [from Eq (3)]

$$x = \int_1^u \left(\frac{dh/du + u}{jE_yA}\right) du \tag{A1}$$

Equation (A1) can be integrated in closed form for numerous functional forms of $h(u)$ and $jE_yA(u)$ The remaining dependent variables are found from $\rho = 1/uA;\ p = p(\rho,h);\ \sigma = \sigma(p,h),\ j^2 = \sigma(\rho u dh/du - u dp/du)du/dx;\ B = (\rho u + dp/du)(du/dx)/j,\ E_y = uB + j/\sigma;\ E_x = Bj/\rho\sigma$, and

$$-\Phi = \int_1^u \frac{udu}{\sigma} + \int_{p_1}^p \frac{dp}{\rho\sigma}$$

The choice of the functional forms of h, jE_yA, and A should be such that the foregoing expressions for j^2 and B are satisfied at $x = 0$

References

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Optimum Geometric Factors for Semicircular Fins in Radiation-Cooled Nozzles

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Radiation heat transfer may enhance the cooling of rocket surfaces The use of semicircular fins attached to a cylindrical rocket body was investigated The fins are comprised of an equal number of concave and convex sections with unequal radii Equations are derived for the fins, radiation geometric factors, and the efficiency of finned surfaces The equations are programmed on the IBM 704 computer, and the results of the calculations are plotted The efficiency of the finned surfaces considered was significantly improved when the ratio of convex fin radius to concave fin radius was chosen as greater than one

Nomenclature

A	= fin area, ft²
A_c ,A_x	= total surface of concave and convex fin area, respectively, ft²
a,b,c,d,e,f	= angle, deg
E_f	= effective (approximate) fin efficiency, dimensionless
F	= geometric factor, dimensionless
F_c ,F_x	= geometric factor for concave and convex fin, respectively, dimensionless
M_1,M_2,M_3,M_4	= slopes of tangents 1, 2, 3, and 4, respectively, dimensionless
Q	= heat dissipated by radiation, Btu/sec
R_p	= outer radius of nonfinned cylinder, ft

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